BMATH FIRST YEAR,, NUMERICAL METHODS, MIDERM EXAMINATION

Answer ANY FIVE questions. Date: 21st February, Friday, 2025. Time: 10 am - 1 pm.

(1) Assume your computer has 64 bits reserved to represent a floating point number where 11 bits are for the exponent, 1 for the sign and 52 bits for the fraction. What is the floating point representation of the fraction $-15/32$ (base 10)? What is the largest number smaller than $-15/32$ that can be represented as a floating point number? (3+2 = 5 marks)

(2) Describe in brief the power method for finding eigenvalues and eigenvectors of matrices. Using the matrix $\begin{pmatrix} 0 & 2 \\ 2 & 3 \end{pmatrix}$ and the inital vector $(1, 1)$ as an example, do 3 iterations to show how this method works. (2+3 = 5 marks)

(3) Describe in brief the QR method to find all eigenvalues of (a large class of) matrices. Prove that any nonsingular matrix has a QR decomposition. (2+3 =5 marks)

(4) What is the least square regression problem? Deduce the least square regression formula using any method of your choice. (2+3 = 5 marks)

(5) What is the interpolation problem? Describe briefly the cubic spline method with three points (x_i, y_i) with $i = 1, 2, 3$ and $x_1 < x_2 < x_3$. (2+3 = 5 marks)

(6) Describe briefly the Newton-Raphson method for finding roots of equations. Can you draw (approximately) the graph of a function) for which the Newton-Raphson method will not work? (3+ 2 = 5 marks)

(7) Describe the forward difference formula and central difference formula for approximating derivatives. Find out the order of the approximation for these two methods. (2+3 = 5 marks)